NAG Toolbox for MATLAB

s18gk

1 Purpose

s18gk returns a sequence of values for the Bessel functions $J_{\alpha+n-1}(z)$ or $J_{\alpha-n+1}(z)$ for complex z, nonnegative $\alpha < 1$ and $n = 1, 2, \ldots, |N| + 1$.

2 Syntax

```
[b, ifail] = s18gk(z, a, nl)
```

3 Description

s18gk evaluates a sequence of values for the Bessel function of the first kind $J_\alpha(z)$, where z is complex and nonzero and α is the order with $0 \leq \alpha < 1$. The $(|N| + 1)$-member sequence is generated for orders $\alpha, \alpha + 1, \ldots, \alpha + |N|$ when $N \geq 0$. Note that $+$ is replaced by $-$ when $N < 0$. For positive orders the function may also be called with $z = 0$, since $J_q(0) = 0$ when $q > 0$. For negative orders the formula

$$J_{-q}(z) = \cos(\pi q) J_q(z) - \sin(\pi q) Y_q(z)$$

is used to generate the required sequence. The appropriate values of $J_q(z)$ and $Y_q(z)$ are obtained by calls to s17de and s17dc.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **z – complex scalar**

The argument z of the function.

Constraint: $\mathbf{z} \neq (0.0, 0.0)$ when $\mathbf{nl} < 0$.

2: **a – double scalar**

The order α of the first member in the required sequence of function values.

Constraint: $0.0 \leq \mathbf{a} < 1.0$.

3: **nl – int32 scalar**

The value of N.

Constraint: $\text{abs}(\mathbf{nl}) \leq 101$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **b**$(*)$ **– complex array**

Note: the dimension of the array **b** must be at least abs(**nl**) + 1.

With **ifail** = 0 or 3, the required sequence of function values: **b**(n) contains $J_{\alpha+n-1}(z)$ if **nl** ≥ 0 and $J_{\alpha-n+1}(z)$ otherwise, for $n = 1, 2, \ldots,$ abs(**nl**) + 1.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **z** = (0.0, 0.0) when **nl** < 0,
or **a** < 0.0,
or **a** ≥ 1.0,
or abs(**nl**) > 101.

ifail = 2

The computation has been abandoned due to the likelihood of overflow.

ifail = 3

The computation has been completed but some precision has been lost.

ifail = 4

The computation has been abandoned due to total loss of precision.

ifail = 5

The computation has been abandoned due to failure to satisfy the termination condition.

7 Accuracy

All constants in s17dc and s17de are specified to approximately 18 digits of precision. If t denotes the number of digits of precision in the floating-point arithmetic being used, then clearly the maximum number of correct digits in the results obtained is limited by $p = \min(t, 18)$. Because of errors in argument reduction when computing elementary functions inside s17dc and s17de, the actual number of correct digits is limited, in general, by $p - s$, where $s \approx \max(1, |\log_{10}|z||, |\log_{10}|\alpha||)$ represents the number of digits lost due to the argument reduction. Thus the larger the values of $|z|$ and $|\alpha|$, the less the precision in the result.

8 Further Comments

None.

9 Example

```
z = complex(0.6, -0.8);
a = 0;
nl = int32(3);
[b, ifail] = s18gk(z, a, nl)
```

```
b =
   1.0565 + 0.2481i
   0.3582 - 0.3754i
  -0.0260 - 0.1254i
  -0.0194 - 0.0086i
ifail =
           0
```